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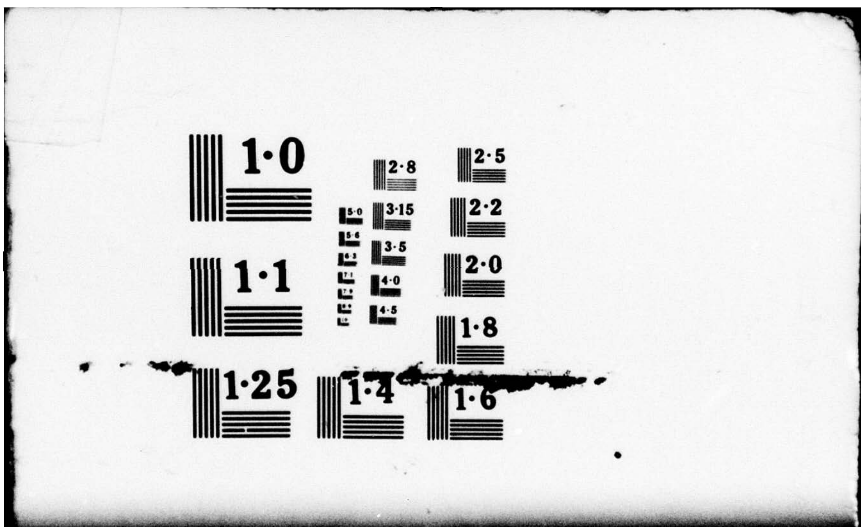
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A NONLINEAR STABILITY APPROACH TO BOUNDARY-LAYER TRANSITION DATA ANALYSIS

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Findings and conclusions of a brief, seven month, effort on fundamental aspects of boundary-layer transition analysis and prediction are reported. Starting from the linear forced response/stability theories of Mack, the capabilities of the attendant maximum allowable linear disturbance amplitude criteria are examined in the context of selected transition experiments. Evidence of distinct nonlinear effects is found in the growth process of internal boundary-layer disturbances having small but finite amplitude heretofore assumed to fall within the realm of linear theory. The flow processes responsible for this behavior are identified,

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the formulation of an attendant nonlinear model differing from classical weakly nonlinear stability theory is set forth, and preliminary predictions, apparently explaining the complex multi-region stability diagram observed in recent experiments by Demetriades, are presented. An important role of nonlinear flow processes and models is suggested for transition data analyses/predictions and an approach to elaboration of such models is outlined.

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I. INTRODUCTION

The present investigation has been motivated by recent flight test analyses (Ref. 1) and parametric studies (Ref. 2) which indicate that the impact dispersion of ballistic reentry vehicles (RV's) may measurably be influenced by asymmetric boundary layer transition (BLT). Whereas the aerodynamic effects induced by asymmetric BLT may be represented in terms of lift and moment coefficient increments (ΔC_{Ltr}, ΔC_{mtr}), the history of such coefficients over the altitude range where BLT advances from the frustrum to the nose of the vehicle is dominated by the dynamics of the underlying process, viz. by the rates of axial progression and circumferential rotation of the asymmetric BLT front as a function of vehicle design and trajectory parameters. Thus, the classical problems of transition prediction and/or control are posed again, now in the aggravated context of three-dimensional flows.

At the present time an unsatisfactory fundamental understanding of the transition process, especially with regard to its late stages of finite amplitude three-dimensional disturbances and subsequent birth of turbulent spots, plagues the researcher as well as the designer concerned with either low-speed or high-speed flows (Refs. 3,4). As a consequence, engineering design practice has relied on empirical correlations of data, generally obtained from small sets of experiments where systematic and controlled variation is achieved only for a few of the numerous dimensionless parameters associated with the equations and the boundary conditions governing the three-dimensional time dependent flows of interest (Ref. 5), while little definition and/or control is provided for the remaining parameters. By

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this process a large number of effects have been exhibited, e.g. Mach number, surface temperature ratio, bluntness, angle of attack, sweep, surface roughness, suction and blowing, the mysterious "unit Reynolds number" etc.; however, no reliable criteria have been evolved for identifying the dominant effect in any one situation and, thus, no means have been achieved for predicting transition Reynolds numbers in even moderately systematic fashion. Based on this long-term experience one may fairly state that only a close coupling between controlled experiments and detailed theoretical modeling of the salient features of the parametric boundary layer response to fully described disturbance environments will produce substantive advances in transition prediction capability. The investigation reported here has largely been keyed to this view point.

To date the theoretical modeling of transition processes has played a distinctly secondary role vis-a-vis the experimental studies and empirical correlations mentioned above. Direct numerical solutions of the three-dimensional time-dependent Navier-Stokes equations for compressible flow could, in principle, describe the desired boundary layer response and its parametric sensitivities; however, their routine execution still lies in the future, particularly for the high Reynolds numbers of interest in practical applications (Refs. 6, 7, 8). Even if these difficulties were surmounted, the physical interpretation of such numerical results in terms of salient linear and non-linear flow instability processes and their parametric sensitivities, as well as the synthesis of attendant transition prediction criteria,would not be straightforward. Thus, the search for asymptotic models, which isolate and describe the dominant processes at various stages of the flow development, appears to provide the most promising

line of fundamental inquiry at the present time. The investigation reported here has been keyed to this view point.

Two approaches broadly falling within the asymptotic model category reside in the use of either turbulence model equations (e.g. Ref. 9), or linear stability and forced response theories (Refs. 10, 11), to approximately describe the flow behavior throughout the stages of linear instability, finite amplitude disturbances and ultimate breakdown into turbulence. The view points basic to these approaches are distinct to such an extent that the potential of the attendant models as fundamental tools for 1) the systematic interpretation of transitional flow processes/sensitivities/effects within the available data base and 2) the theoretically justified extrapolation of those processes/effects to conditions outside the data base, may be gleaned a priori upon a brief comparative discussion.

The turbulence model equations espouse the traditional long-time average view of the unsteady transitional and turbulent flows. This view is embodied in a number of closure coefficients, which essentially represent correlations associated with the disturbance spectrum of the considered flow. Whereas the values of many of the coefficients are chosen by numerical experimentation, i.e. by forcing agreement between numerical predictions and <u>selected</u> experimental data, the approach is at best interpolative and conceptually subject to the same limitations which have plagued the large body of empirical correlations extracted from experiments in the past. In fact, recent experience has provided several examples of unreliable extrapolation, e.g. the failure to predict compressibility effect in two-dimensional turbulent mixing layers (Ref. 12) and to extend results from axisymmetric to two-dimensional incompressible jets (Ref. 13). This experience complements

the mounting body of experimental evidence to the effect that transports in turbulent shear flows are controlled by large coherent flow structures, i.e. typical and recognizable concentrations of transverse mean vorticity at the largest flow scale, whose characteristics differ from one type of flow to another and, for a given flow type, vary as a function of the associated parameters, e.g. Mach number (Ref. 14). These structures presumably represent specific families of deterministic non-linear solutions to the flow equations, viz. wave-like solutions which retain phase information at the largest scale (of the structures), and introduce Reynolds averaging at the lower structural level of fluctuations measured with respect to ensemble averages of structure realizations. Whereas the solutions in question must evolve from, and in the linear limit reduce to, the solutions provided by the forced response and stability theories of Refs. 10 and 11, the latter appear to possess a dintinct conceptual edge over the turbulence model equations as a fundamental tool for systematic transition studies. This is not to say that turbulence model equations cannot be adapted in the future to provide useful working tools on a flow by flow basis. However, this upgrading seems contingent upon the systematic parametric determination of closure coefficients consistent with experiments and large scale structure solutions provided by the linear forced response/stability theories and their extensions to the non-linear regime.

Based on the above premises the investigation reported here has been keyed to the forced response/stability view point of Refs. 10 and 11 with specific attention to the capability of the attendant maximum allowable linear disturbance amplitude criteria to provide systematic transition predictions. In that connection selected recent transition experiments

have been reviewed and, by that process, evidence has been found of distinct non-linear effects in the growth of internal boundary layer disturbances having small but finite amplitudes [e.g. $(u'/U_1) \simeq 10^{-2}$] which, heretofore, had been assumed to fall within the realm of linear theory (Section II). The flow processes responsible for this behavior have then been sought, the formulation of an attendant non-linear model differing from classical weakly non-linear stability theory has been instituted, and preliminary predictions of the model have been obtained, apparently explaining the complex multi-region stability diagram which has been observed in the experiments of Refs. 15 and 16, but has not been predicted by available results of linear theory (Section III). On this basis, preliminary conclusions have been evolved as to the role, nature and modeling of processes required for the fundamental understanding of transition and for the development of attendant systematic prediction criteria (Section IV).

II. DISCUSSION OF SELECTED TRANSITION EXPERIMENTS VIS-A-VIS LINEAR AND WEAKLY NON-LINEAR THEORIES

Recent boundary layer transition experiments at supersonic velocities can be divided into two groups, namely: 1) those where only the transition location is determined on the basis of some experiment specific observable criterion (e.g. inspection of either boundary layer shadowgraphs as in Refs. 18, 19, 20, or wall heat flux measurements as in Ref. 21), and 2) those where the mean flow and surface data are supplemented by hot wire anemometer measurements of the disturbances within the boundary layer as they evolve upstream of, and through, transition (e.g. Refs. 15, 16, 17).

Although the experiments in the first group have yielded a number of useful practical results, e.g. the first cut description of transition zone circumferential asymmetry on pointed slender cones as a function of dimensionless angle of attack (α/θ_c) suggested in Ref. 19, the interplay between nose bluntness and angle of attack in controlling the wind-fixed transition zone asymmetry on spherically capped cones displayed in Ref. 21, the possible synergetic effect of asymmetric body-fixed small-scale roughness and angle of attack in promoting transition on the lee (wind) side of sharp (blunt) nosed vehicles observed in Ref. 21 etc, they have failed to provide mechanistic explanations for the observed effects with attendant rules for a realistic extrapolation of the results from laboratory to flight conditions and disturbance environments. In fact, among the many transition dependancies exhibited in the ground experiments, that associated with the unit Reynolds number predominates and remains most obscure. Whereas the correlation of transition Reynolds

number with unit Reynolds number in supersonic and hypersonic wind tunnels may largely be associated with noise pressure fluctuations radiated onto the model from turbulent tunnel nozzle boundary layers (Ref. 22), the reason for its existence in ballistic ranges remains to be explained, especially since, according to Ref. 19, only a secondary role may be attached to those features of free flight experimentation suspected to influence the results (viz. oscillatory motion and angle of attack, surface roughness, model vibration, non-uniform model surface temperature). In view of 1) the limited understanding of the competing roles played by the many parameters entering the problem, and 2) the impossibility of complete simulation in ground facilities, the resolution of the open questions noted above and the development of reliable transition prediction criteria then require reconsideration of the data in the context of a unified physico-mathematical model of the underlying flow processes. Clearly the predictions of the model must be validated against the available base of fully characterized and controlled experiments. The discussions below indicate that the results of group 2 provide the most fruitful testing grounds for that purpose.

As stated in the introduction, the linear forced response/stability theories of Mack (Refs. 10, 11) provide a promising theoretical framework for systematic data analysis. Their use, however, must be conditioned by a judicious appreciation/assessment of their less developed aspects, specifically: a) the current lack of a theory capable of predicting in all practical instances the initial amplitude of each Fourier component of the internal disturbances forced by the prevailing external disturbances, and b) the general adequacy of transition criteria based on a

threshold level for the amplitude of the most amplified single frequency linear internal disturbance. These aspects can hardly be elucidated by applications of the theory to situations where the transition Reynolds number changes mainly because either the mean boundary layer or the spectrum of external disturbances or one of the boundary conditions (e.g. the wall temperature ratio) change. Applicability of the theory to such situations has already been exhibited by Mack (Refs. 10, 11). Accordingly, the first step in the research reported here was to select within the chosen base (Refs. 15 through 21) those data which promised potentially new tests.

Cursory analysis indicates that the experiments in group 1 do not readily provide the desired new tests. Specifically, the few available "exact" results for the supersonic laminar boundary layer over a pointed cone at incidence (Ref. 23) suggest that the circumferential asymmetry of the transition zone on pointed slender cones at angle of attack is dominated by the attendant asymmetry in laminar boundary layer thickness, at least over the range $(\alpha/\theta_c) \lesssim .2$ where the layer maintains self-preserving structure. For $(\alpha/\theta_c) \gtrsim .2$ the boundary layer profiles and thickness depart from self-preserving behavior in a region of increasing circumferential extent about the leeward generator; reduced disturbance amplification rates appear to be associated with those profiles leading to reduced rates in the forward shift of the asymmetric transition front at increasing angles of attack, as indicated by the correlation of Ref. 19. For blunted cones at incidence the additional effect of entropy swallowing must be considered, with the realization that the inviscid cross-flow yields an effective increase (decrease) in the rate of swallowing on the

windward (leeward) side and, thus, induces a destabilizing* (stabilizing) effect opposite to that due to the simultaneous thinning (thickening) of the boundary layer. Whereas entropy swallowing was far from complete in those experiments of Ref. 21 where transition was observed first on the windward meridian, the reversal in transition front asymmetry with increasing bluntness may then be attributed to a predominant effect of differential entropy swallowing on the wind and lee sides. Unfortunately, codes currently available do not permit a straightforward calculation of the swallowing process in three-dimensional flows. Thus, the observed interplay of bluntness and angle of attack in controlling the asymmetry of the transition front on bodies at incidence does not provide at present a direct unequivocal test for a theoretical model. Lack of information about the external disturbances and their coupling to the boundary layer causes the same negative conclusion to be reached with regard to the unit Reynolds number and wall temperature ratio effects exhibited in Refs. 19 and 20. Similarly, the tentative state of affairs in modeling roughness effects by way of phenomenological mean profile perturbations and attendant changes in the stability/amplification of internal disturbances (e.g. Ref. 24) contaminates a possible test of the theory against the relevant observations in Ref. 21. The quest for direct model assessment should then be shifted to the experiments of group 2, albeit with a commitment to tentatively reconsider the strong unit Reynolds number effect

* For bodies at zero incidence under fixed flight/ambient conditions, it is known that transition moves rearward if the nosetip radius is increased, and the degree of rearward movement may be correlated with the entropy swallowing distance

manifested by firing range experiments in the light of possible model extensions suggested by that assessment.

The most prominent aspects of Demetriades' observations (Refs. 15, 16) perhaps reside in 1) the apparent non-linear nature of the oscillation associated with the "laminar waves" which become prominent about half way to the station where mean flow and surface response sensing methods indicate onset of transition, and 2) the peaking of the laminar waves prior to transition onset under conditions representative of approach/crossing of the upper neutral branch. These aspects run counter some premises of the linear forced response/stability approach, namely: 1) non-linearity occurs only in a small region immediately preceding transition, 2) criteria based on a threshold amplitude for linear disturbances reasonably predict transition onset because the process occurs in regions of rapid growth for those disturbances. The apparent divergence clearly warrants more detailed scrutiny.

The aforenoted aspects of Demetriades' results are readily apparent upon a three-dimensional view of the evolution of spectra of the hot wire signals such as those displayed in Fig. 1 for the TBLC measurements (Ref. 15) in the boundary layer over a sharp-tipped 5-degree half-angle cone at zero incidence with free stream Mach number $M_\infty = 8$, edge Mach number $M_e = 6.79$, tunnel supply temperature $T_o = 755^oK$, supply pressure $p_o = 150$ and 350 psia, unit Reynolds number based on edge conditions $Re'_e = 5.5 \times 10^4$ and 8.5×10^4 cm^{-1}, surface-to-stagnation temperature ratio $(T_w/T_o) = 0.75$. For each Reynolds number Re_{ex} (based on edge conditions) Fig. 1 displays the spectra recorded at (y/δ) intervals of 0.08; those measured nearest the wall, i.e. $(y/\delta) = 0.12$, are plotted on the far side of the diagram,

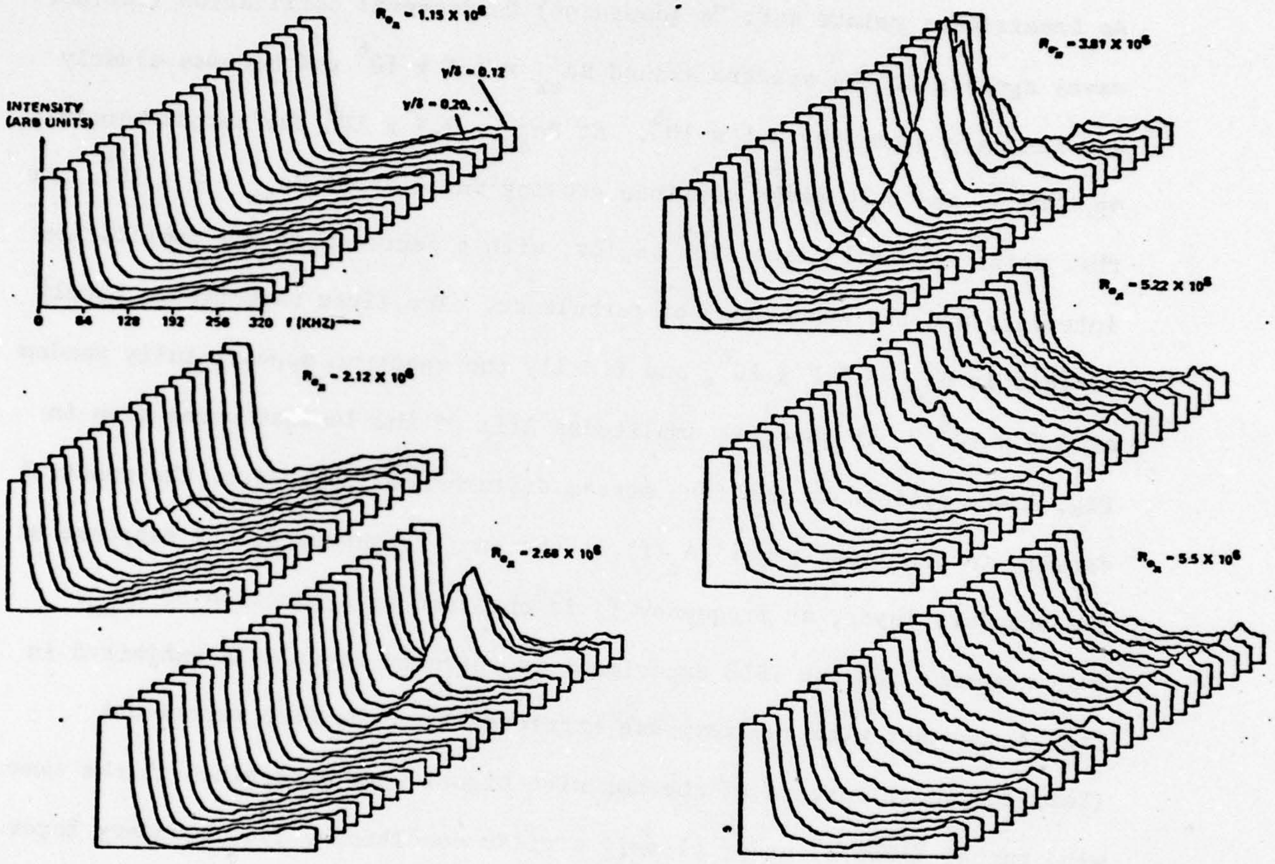

Fig. 1 Three-dimensional view of the spectra of hot wire signals recorded in the transitional boundary layer over a pointed slender cone at Mach number 7. From Reference 15.

while those outside the boundary layer are presented nearest the reader.*
As Demetriades points out: "a (dominant) fundamental oscillation (laminar wave) appears in the spectra around $Re_{ex} = 1.5 \times 10^6$ and becomes clearly and strongly formed by 2.5×10^6. At $Re_{ex} = 2.7 \times 10^6$ the second harmonic appears**, and both peaks continue growing until about $Re_{ex} = 4 \times 10^6$. At that point spectral dispersion begins, with a decrease of the oscillation intensity and the appearance of turbulence. The first harmonic is barely visible at $Re_{ex} = 5.2 \times 10^6$, and finally the spectrum appears fully random at 5.5×10^6." If the peak amplitudes A(f) of the laminar waves seen in Fig. 1 are ratioed to the free stream disturbance input (i.e. the spectral density to the first power) $A_o(f)$ at the same frequency f, the response of the boundary layer, at frequency f, is obtained as a function of Re_{ex}. Such response for the TBLC experiment (spectra of Fig. 1) is exhibited in Fig. 2 together with the response extracted from the BLTB experiment (Ref. 16), where spectra of the hot wire signal were determined in the same wind tunnel (AEDC/B) under closely similar conditions, i.e. boundary layer over a sharp-tipped 4-degree half-angle cone at zero incidence, free stream Mach number $M_\infty = 8$, edge Mach number $M_e = 7$, supply temperature $T_o = 728^oK$, supply pressure p_o = 106 and 160 psia, unit Reynolds number $Re_e' = 1.66 \times 10^4$ and 2.5×10^4 cm^{-1}, surface-to-stagnation temperature ratio $(T_w/T_o) = 0.41$ and 0.80. Since the free stream disturbance spectra remained quite similar

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* The spectra at $Re_{ex} \leq 2.7 \times 10^6$ pertain to the experiments at p_o = 150 psia, while those at larger Re_{ex} pertain to p_o = 350 psia.

** Demetriates indicates $Re_{ex} = 3.5 \times 10^6$ for this event; however, the data of Fig. 1 as well as those in Fig. 9 of Ref. 15 suggest the presence of a distinct second harmonic already at $Re_{ex} \simeq 2.7 \times 10^6$.

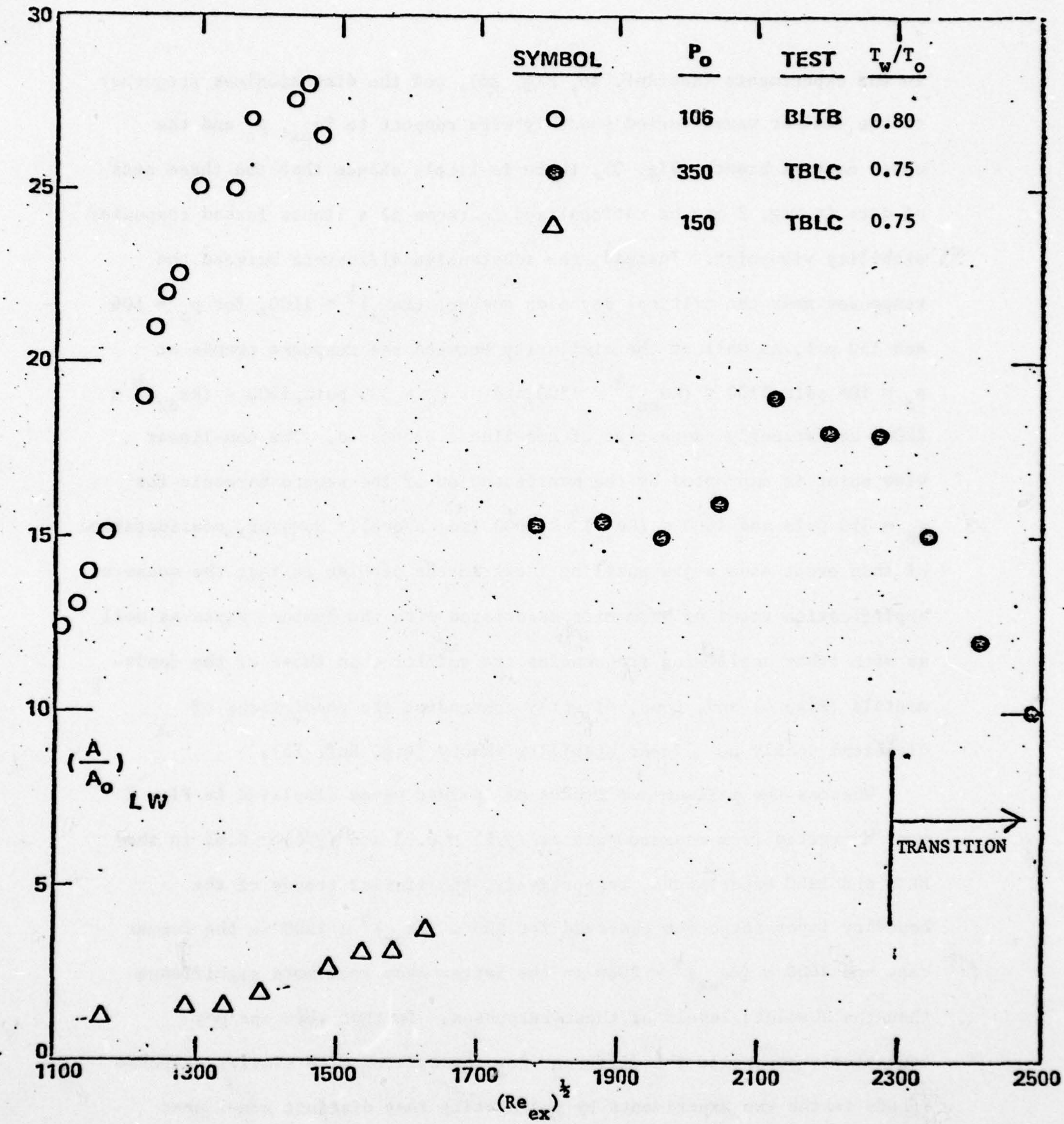

Fig. 2 BLTB and TBLC experiments. Measured maximum amplitude of laminar waves scaled to spectral density of freestream disturbances. From Reference 16.

in the experiments (see Ref. 16, Fig. 26), and the dimensionless frequency of the laminar waves varied smoothly with respect to Re_{ex}, p_o and the upper neutral branch (Fig. 3), there is little chance that the three sets of data in Fig. 2 may be rationalized in terms of a linear forced response/ stability viewpoint. Instead, the substantive difference between the responses near the critical Reynolds number, $(Re_{ex})^{\frac{1}{2}} \simeq 1100$, for $p_o = 106$ and 150 psi, as well as the similarity between the response trends at $p_o = 106$ psia, $1100 \leq (Re_{ex})^{\frac{1}{2}} \leq 1500$, and at $p_o = 350$ psia, $1600 \leq (Re_{ex})^{\frac{1}{2}} \leq 2000$, are strongly suggestive of non-linear processes. The non-linear view point is supported by the manifestation of the second harmonic for $p_o = 350$ psia and $1600 \leq (Re_{ex})^{\frac{1}{2}} \leq 2000$ (see above). However, consideration of this event adds a new puzzling facet to the problem in that the measured amplification rates of harmonics associated with the laminar waves as well as with other amplifying frequencies are smaller than those of the fundamentals (Fig. 4) and, thus, directly contradict the predictions of classical weakly non-linear stability theory (e.g. Ref. 25).

Whereas the maximum amplitudes of laminar waves displayed in Fig. 2 were extracted from measurements at $(y/\delta) \simeq 0.73$ and $(y/\delta) \simeq 0.65$ in the BLTB and TBLC experiments, respectively, the similar trends of the boundary layer responses observed for $100 \leq (Re_{ex})^{\frac{1}{2}} \leq 1500$ in the former case and $1600 \leq (Re_{ex})^{\frac{1}{2}} \leq 2000$ in the latter case seem more significant than the absolute levels of those responses. In that vein one may tentatively reconcile the different Re_{ex} associated with similar response trends in the two experiments by postulating that distinct non-linear effects are largely confined to a limited region whose lateral extent about the critical layer $(y/\delta) \simeq .85$ increases with Re_{ex} so as to

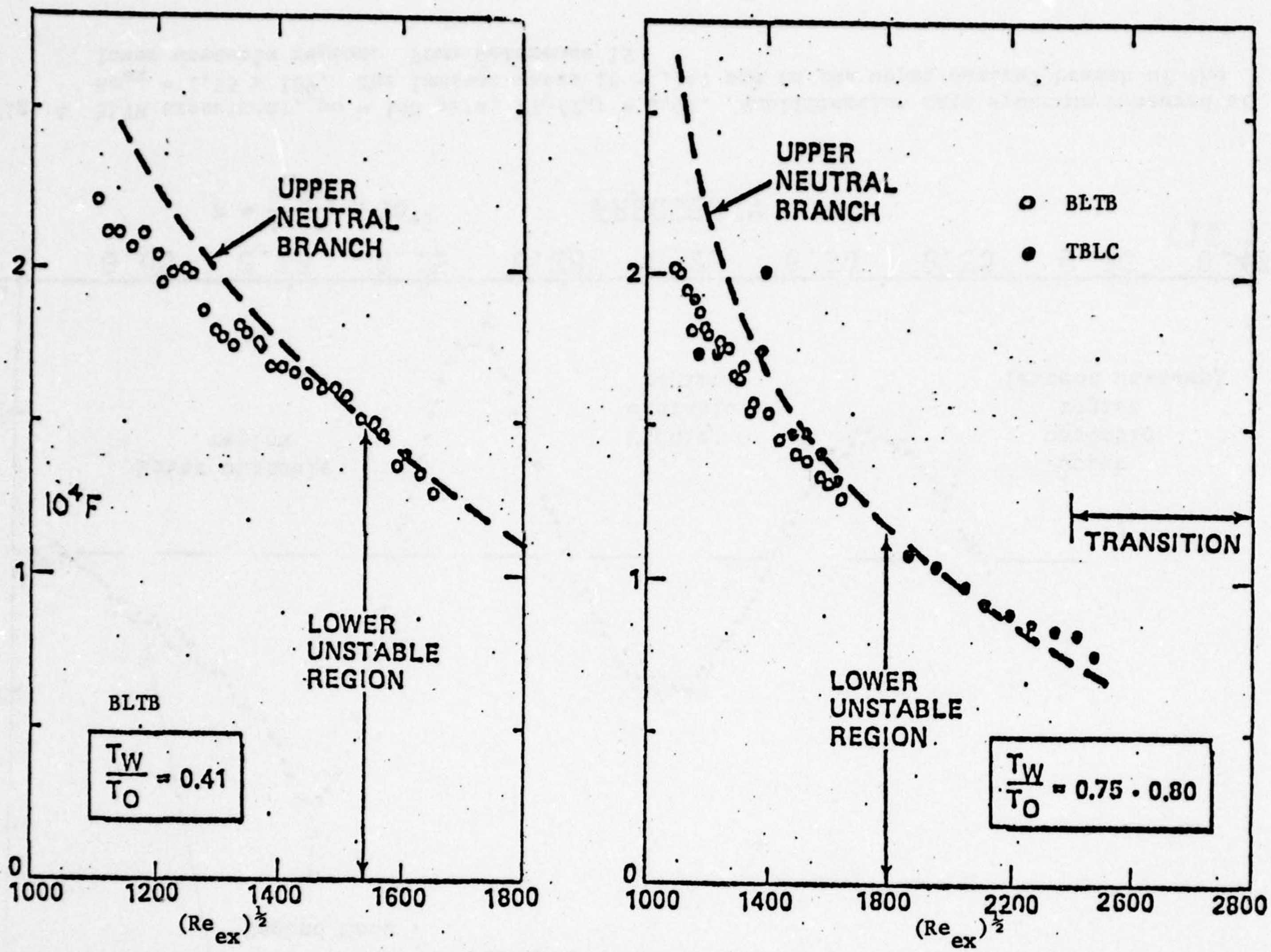

Fig. 3 BLTB and TBLC experiments. Measured dimensionless frequency of the laminar waves as a function of $(Re_{ex})^{\frac{1}{2}}$, and its relation to the upper neutral branch of the lower unstable region. From Reference 16.

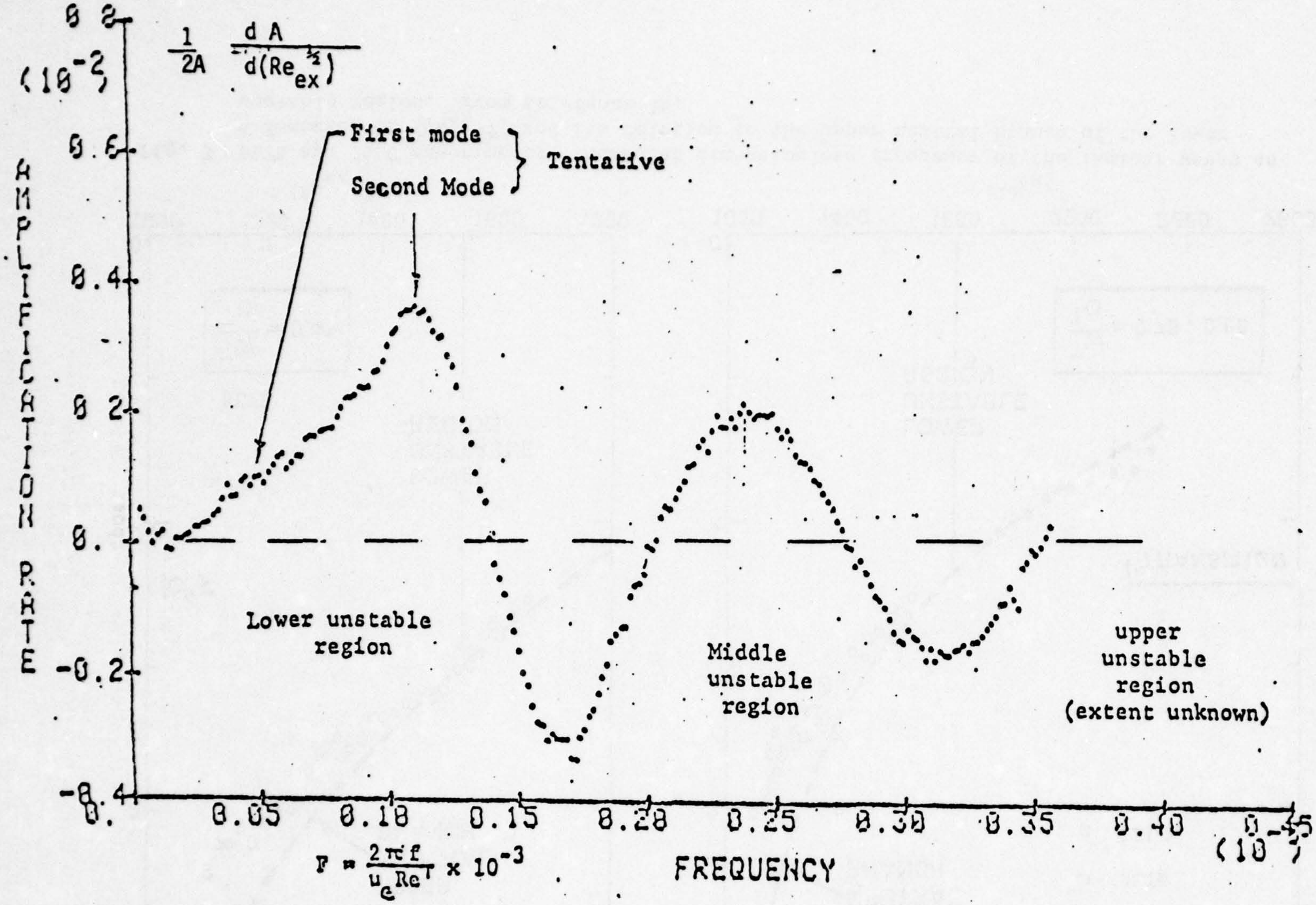

Fig. 4 BLTB experiment, po = 160 psia, $(T_w/T_o) = 0.41$. Amplification rate spectrum measured at $Re_{ex} = 2.55 \times 10^6$. The laminar waves ($F \simeq .14$) are on the upper neutral branch of the lower unstable region. From Reference 16

encompass $(y/\delta) \simeq .73$ at $Re_{ex}^{\frac{1}{2}} \gtrsim 1100$ and $(y/\delta) \simeq .65$ at $Re_{ex}^{\frac{1}{2}} \gtrsim 1600$. Although broadly consistent with the spectra shown in Fig. 1 for $2.12 \times 10^6 \leq Re_{ex} \leq 3.81 \times 10^6$, this conjectured behavior departs so markedly from classical weakly non-linear theory as to warrant further scrutiny in the light of 1) the available experiments and 2) the formulation of an attendant theoretical model which also accounts for the apparently anomalous growth rates of harmonics noted above. Since the data reported by Demetriades pertain to a single position (y/δ) across the boundary layer, this scrutiny, pursued in the remainder of this section and through the following section of the present report, must, however, fall back on the low speed transition data of Klebanoff et al. (Ref. 26).

The low-speed experiments of Ref. 26 forcibly demonstrate that Tollmien-Schlichting waves evolve into turbulence through a definite and reproducible sequence of events, which comprise several stages, viz.: a primary state governed by the two-dimensional linear stability theory, a second stage of finite amplitude disturbances where strong three-dimensional effects are observed, and, finally, a third stage involving the birth of turbulent spots. Of specific interest here are the qualitative features of the non-linear three-dimensional wave motion, which controls the flow behavior in the second stage and, in the process, develops the conditions prerequisite to the instability for the third stage breakdown into turbulence. These features have generally been examined in the context of, and deemed consistent with, Benney's model (Ref. 25) for the weakly non-linear superposition of a two-dimensional Tollmien-Schlichting wave and a phase-locked three-dimensional wave

having a periodic spanwise variation in wave amplitude. In view of the unresolved questions posed by Demetriades' measurements a close comparative reexamination of the low-speed theory and experiments is in order to ascertain whether any discrepancies exist which may be resolved by a modified theoretical view point accomodating the available evidence about non-linear flow processes at high as well as low Mach numbers.

In Benney's model a purely two-dimensional temporally growing oscillation $\mu[\hat{U}_1(y), \hat{V}_1(y)] \exp[i\alpha(x-ct)]$ is compounded with a temporally growing standing wave in the spanwise direction, $\lambda[\hat{u}_1(y) \cos \beta z, \hat{v}_1(y) \cos \beta z, \hat{w}_1(y) \sin \beta z] \exp[i\alpha(x-ct)]$, to yield a mean and an oscillatory secondary flows. For cases involving either a fluid in the presence of a solid boundary or a fluid extending to infinity, the mean secondary flow is characterized by velocity components

$$v_o^{(2)}(y,z,t) = [\lambda^2 \hat{v}_a(y) \cos 2\beta z + \lambda \mu \hat{v}_b(y) \cos \beta z] [\exp(2\alpha c_i t)-1] [2\alpha c_i]^{-1} \tag{1-a}$$

$$w_o^{(2)}(y,z,t) = [\lambda^2 \hat{w}_a(y) \sin 2\beta z + \lambda \mu \hat{w}_b(y) \sin \beta z] [\exp(2\alpha c_i t)-1] [2\alpha c_i]^{-1} \tag{1-b}$$

and the oscillatory flow by velocity fluctuations

$$v_2^{(2)}(x,y,z,t) = [\lambda^2 \hat{v}_e(y) \cos 2\beta z + \lambda \mu \hat{v}_f(y) \cos \beta z + \lambda^2 \hat{v}_g(y) + \mu^2 \hat{v}_h(y)] \exp[2i\alpha(x-ct)] \tag{2-a}$$

$$w_2^{(2)}(x,y,z,t) = [\lambda^2 \hat{w}_e(y) \sin 2\beta z + \lambda \mu \hat{w}_f(y) \sin \beta z +$$

$$\lambda^2 \hat{w}_g(y) + \mu^2 \hat{w}_h(y)] \exp [2i\alpha (x-ct)] \qquad (2\text{-b})$$

which are bound with the fundamental, having the same phase velocity but half the wave length.

The coherence of fundamental and "secondary" oscillations is well reproduced in the experimental studies of controlled amplifying three-dimensional disturbances with spanwise amplitude modulation (Fig. 5, Ref. 26). Although the behavior characteristic of the finite amplitude region, i.e. a very rapid growth of rms fluctuations at a peak and an initially slow growth at a valley compared to linear theory, arises suddenly and at surprisingly low disturbance levels [e.g. $(u'/U_1) \simeq 10^{-2}$ in Fig. 6], the wave phase angle in the downstream direction shows no significant departure of wave velocity from the linear value and measured distributions of phase across the boundary layer (relative to that observed at the outer edge) show no substantial change, until just before breakdown occurs at a peak. However, several discrepancies between model and observations become apparent when the evolution of the non-linear flow is examined in detail. Specifically, measured distributions of r.m.s. u'-fluctuations (associated with the fundamental as well as the secondary oscillations) and spanwise W-component of mean velocity (associated only with the mean secondary flow) indicate different apparent growth rates at different locations (y/δ) across the boundary layer (Figs. 7,8). Also the evolving non-linear effects appear to influence a region of increasing dimensionless transversal extent (y/δ) (Fig. 7).

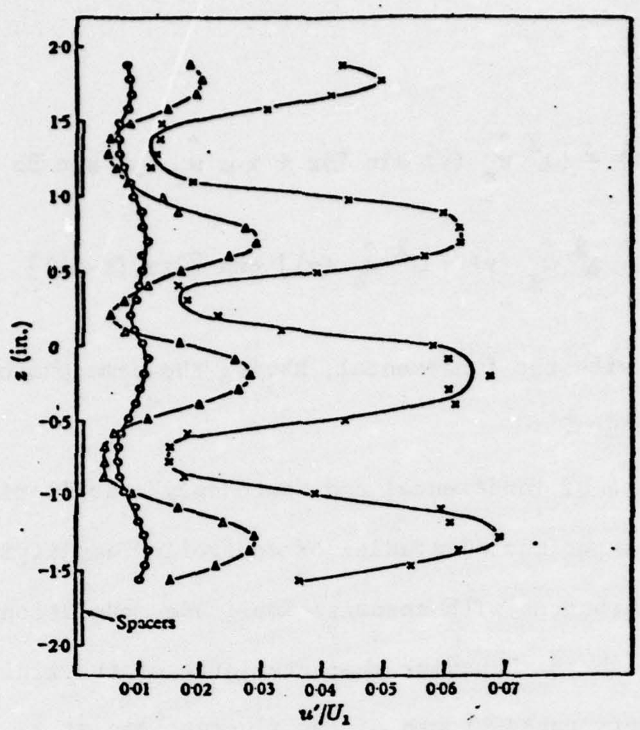

Fig. 5 Experiments of Reference 26. Spanwise distributions of rms u'-fluctuations at different distances downstream from vibrating ribbon: 145 c/s wave, $y = 0.042$ in, $U_1/\nu = 3.1 \times 10^5 \mathrm{ft.}^{-1}$. o, $(x-x_o) = 1$ in.; Δ, $(x-x_o) = 4$ in.; x, $(x-x_o) = 5.5$ in.

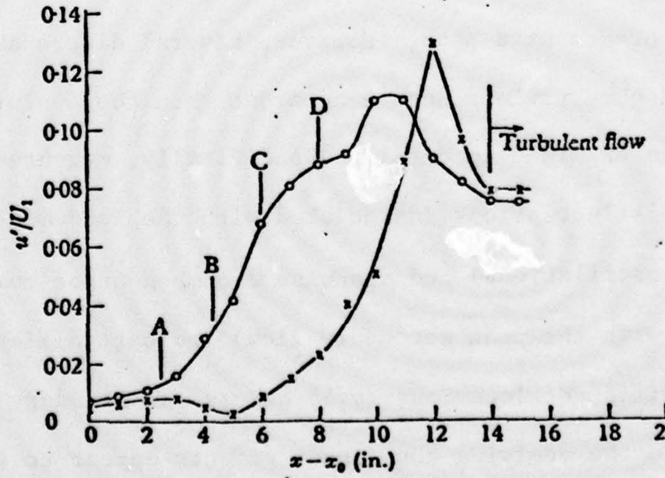

Fig. 6 Experiments of Reference 26. Rms u'-fluctuations at peak and valley for $y = 0.042$ in. and input disturbance amplitudes $(u_o'/U_1) \simeq .007$ and .005 at peak and valley, respectively. o, $z = -0.2$ in, (peak); x, $z = -0.75$ in. (valley).

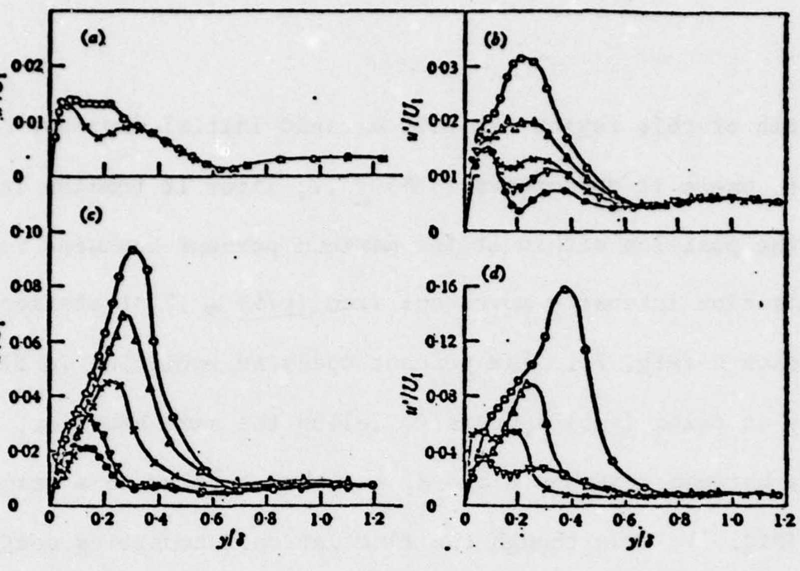

Fig. 7 Experiments of Reference 26. Distribution of rms u'-fluctuations across boundary layer: 145 c/s wave, $U_1/\nu = 3.1 \times 10^5$ ft.$^{-1}$. (a) Station A: o,z = - 0.2 in., x,z = - 0.75 in. (b) Station B: o,z = - 0.2 in.; Δ,z = - 0.35 in; x,z = - 0.45 in.; ∇,z = - 0.55 in.; ●,z = - 0.65 in.; (c) Station C; (d) Station D.

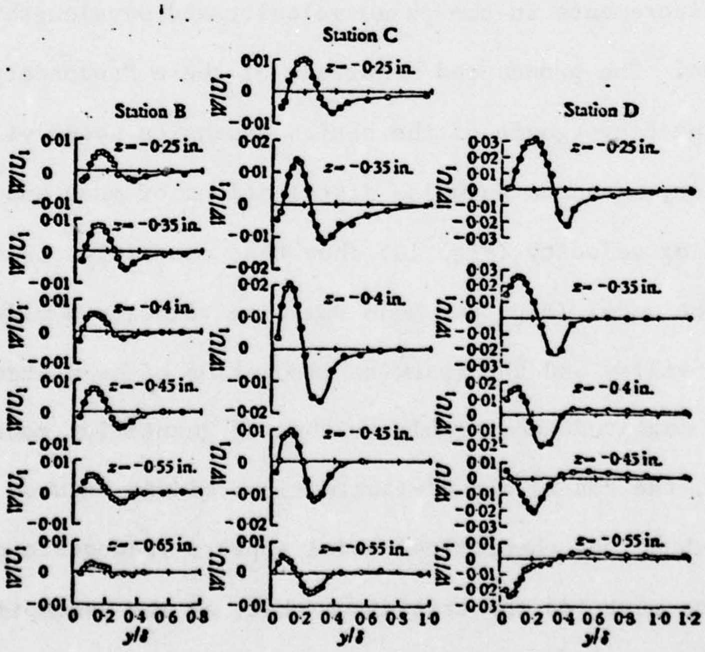

Fig. 8 Experiments of Reference 26. Distributions of spanwise component of mean velocity across boundary layer: 145 c/s wave, $U_1/\nu = 3.1 \times 10^5$ ft.$^{-1}$

The width of this region exhibits a rapid initial increase between stations A and B, where it encompases $(y/\delta) \lesssim .7$; later it remains largely unchanged, while the position within it for maximum percent spanwise variation of u'-fluctuation intensity moves out from $(y/\delta) \approx .2$ at station B to $(y/\delta) \approx .4$ at station D (Fig. 7). The percent spanwise variation in fluctuation intensity at fixed (y/δ) appears to follow the same behavior, i.e. it increases rapidly between stations A and B, but then settles to a reasonably constant level (Fig. 5), even though the fluctuation intensities continue to increase (Fig. 7). By contrast, the (y/δ) position for maximum percent spanwise variation of fluctuation intensity changes only downstream of station B, in rough synchronism with the shift of the critical layer due to "secondary" distortions in mean streamwise velocity profiles (Fig. 9) and attendant increments in the phase velocity and wavelength of the basic disturbance. The pronounced magnitude of these "secondary" shifts casts doubt on the convergence of the series expansion underlying Benney's model and, in fact, measured spanwise distributions of mean and fluctuating components of velocity (Fig. 10) show that quantities considered of second order in the model (e.g. the mean spanwise velocity W midway between a peak and a valley and the spanwise modulation of mean streamwise velocity U) achieve magnitude comparable to that of quantities considered of first order (e.g. the rms w' and u'-fluctuations midway between a peak and a valley). Thus, the theoretical model appears inadequate on several counts. To these one must add the criticisms that: a) the assumption of equal phase velocity for the fundamental two-dimensional and three-dimensional waves cannot be satisfied by the well established linear stability eigensolutions for Blasius and Falkner-Skan profiles; b) if that assumption is relaxed,

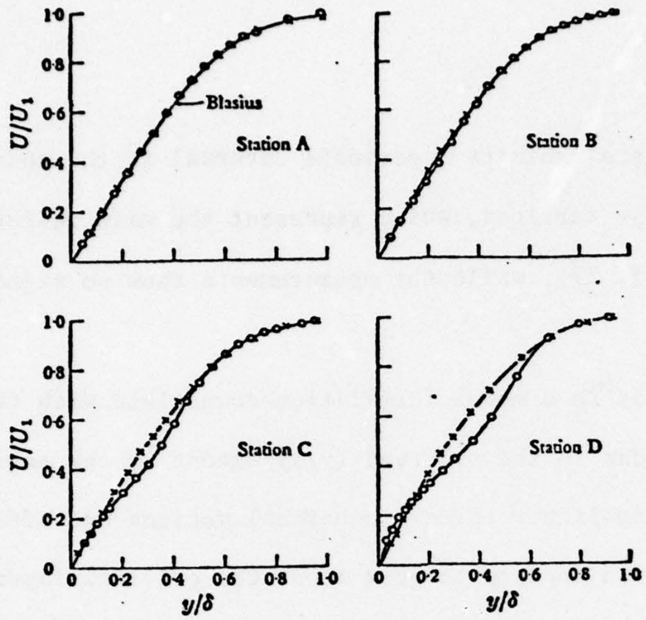

Fig. 9 Experiments of Reference 26. Mean velocity distributions across boundary layer at peak (o) and valley (x): 145 c/s wave, $U_1/\nu = 3.1 \times 10^5$ ft^{-1}.

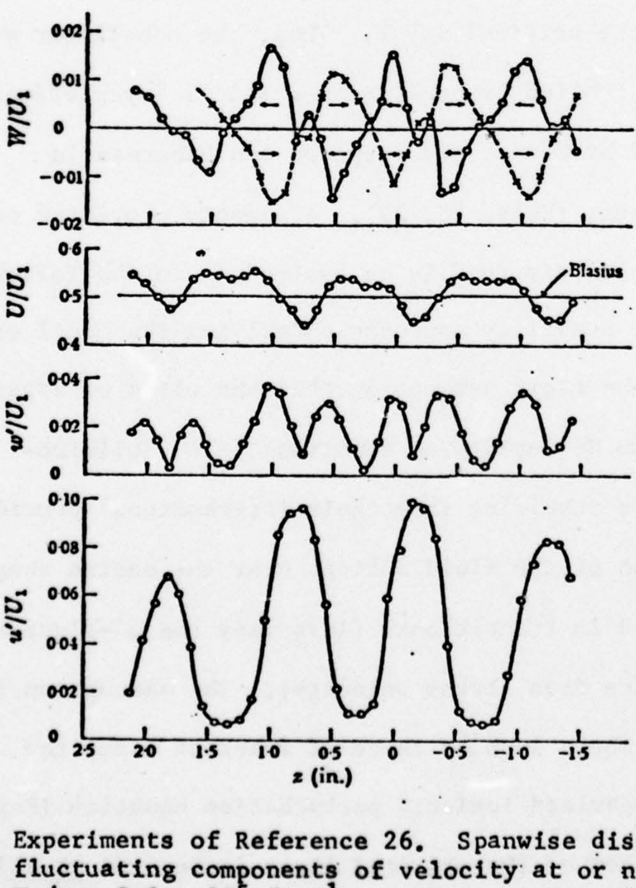

Fig. 10 Experiments of Reference 26. Spanwise distributions of mean and fluctuating components of velocity at or near Station C: 145 c/s wave, $U_1/\nu = 3.1 \times 10^5$ ft.$^{-1}$. ———, $y = 0.31\delta$; ----, $y = 0.11\nu$

the theoretical predictions exhibits a periodic reversal in the sense of rotation of the streamwise vortices, which represent the main feature of the secondary motion (Ref. 27), while the measurements show no significant phase shift.

The strongest clue as to a model formulation compatible with the experimental evidence resides in the observed (y/δ) extent of the region characterized by large non-linear three-dimensional motions and effects. As noted above, this region appears to grow about the critical layer, in rough synchronism with the growth in the spanwise variation of u'-fluctuation intensity; meanwhile the (y/δ) position for maximum percent spanwise variation in fluctuation intensity within the region approximately tracks the shifting position of the critical layer. Thus, the non-linear wave behavior appears to be controlled by non-linear critical layer effects, analogous to those studied by Benney and Bergeron and Haberman in connection with neutral waves (Refs. 28, 29). Arguments providing conceptual support for this view may readily be evolved as in the following.

The weakly non-linear stability approach underlying the model of Ref. 25 is predicated on the tacit assumption that the class of attendant asymptotic solutions to the Navier-Stokes equations (i.e. Tollmien-Schlichting waves and waves resulting from their interactions) provide a uniformly valid description of the fluid motions over the entire range of wave amplitudes encountered in transitional flows (say rms u'-fluctuations $\leq$ 15 to 20% of the reference free stream velocity). The assumption implies that, for high Reynolds numbers such as those of interest here, the singular nature of the linearized inviscid perturbation equation (Rayleigh equation) in the neighborhood of the critical layer is removed <u>at all wave</u>

amplitudes by the sole effect of vorticity diffusion as embodied in the linearized viscous perturbation equation (Orr-Sommerfeld equation). This predominant effect of viscosity independent of wave amplitude, however, runs counter the substantive experience accumulated with singular perturbation problems. As pointed out by Benney and coworkers (Refs. 28, 29), one expects that, depending upon wave amplitude, the singularity of the linearized inviscid perturbation equation is removed by either the effect of the small linear viscous operator or the effect of the small non-linear operators descriptive of non-linear vorticity convection in two-dimensional wave motions and vorticity convection as well as stretching in three-dimensional wave motions. Whereas the viscous effect is confined to a (boundary) region about the critical layer having thickness independent of wave amplitude and proportional to $2(\alpha \, Re_\delta u'_c)^{-1/3}$ (where Re_δ denotes the Reynolds number of the mean flow, α the dimensionless wave number of the disturbance , and u'_c the dimensionless mean velocity gradient at the critical layer), while the non-linear effects influence the flow in a region about the critical layer having thickness proportional to $(A/u'_c)^{\frac{1}{2}}$ (where A denotes the disturbance amplitude), there is always a wave amplitude A_c wherefor the boundary layer scale associated with non-linear effects becomes larger than the scale associated with the viscous effect. For $A \gtrsim A_c$ the Orr-Sommerfeld description of the wave motion near the critical layer ceases to be valid, and a new class of asymptotic solutions to the Navier Stokes equations, recognizing the non-linear vorticity convection and stretching, are required to describe the subsequent growth and/or decay of the wave. Interestingly enough for the controlled transition experiments of Ref. 26 one has: unit Reynolds number $(U_1/\nu) = 10^4 cm^{-1}$,

boundary layer thickness $\delta \simeq .5$ cm, dimensionless wave number of the fundamental disturbance $\alpha \simeq .9$, dimensionless mean (Blasius) velocity gradient at the critical layer $u_c' \simeq 1.6$, and, thus $A \simeq 4(u_c')^{1/3}(\alpha Re_\delta)^{-2/3}$ $\simeq .017$, in reasonable agreement with the rms u'-fluctuation measured at the station where the onset of non-linearity is manifested (Fig. 6). Thus, the entire second and thrid stages of transition fall within the domain of wave growth controlled by non-linear critical layer effects.

Further progress in the analysis of the non-linear flow processes discussed above can only be achieved upon development and validation of a model descriptive of non-linear critical layer effects in amplifying and equilibrating waves. The motivation for such development and validation, undertaken in Section III of the report, is manifold and transcends the scientific objective of realistically modeling the observed non-linear stages of transition.

Admittedly the experiments of Refs. 15, 16 and 26 tend to overemphasize the role of the non-linear stage within the overall transition process because of the high external disturbance levels imposed on the boundary layer. In many practical situations, depending on mean flow conditions as well as on the nature/level/spectrum of external disturbances, a considerable length of linear disturbance growth may precede and dominate the length occupied by the non-linear stages of transition. This is typically the case in low speed, low-free-stream-turbulence experiments, where disturbances undergo very large amplification prior to transition (Ref. 3). The linear view point of Mack would seem adequate for these cases, although the conclusion may be tempered by the

mixed success achieved by the linear theories in predicting transition on low drag hydrodynamic configurations. As the flow Mach number increases, the "equivalent" linear disturbance amplification to transition decreases markedly (Ref. 10), and the length of the non-linear region becomes a measurable fraction of the transition distance. More important, the expedient of letting the linear disturbance grow to some arbitrary level acquires a distinctly ad hoc flavor. As Mack points out in the Appendix to Ref. 10, the magnitudes of the transition Reynolds numbers observed on flat plates and cones exposed to supersonic flows cannot be reconciled in terms of linear stability theory together with the known exact transformation between the laminar boundary layer equations/solutions in the two cases and the approximate transformation of the attendant linearized stability equations. Whereas the observed end-of-transition Reynolds numbers on cones Re_{tc} always exceed those on flat plates and hollow cylinders Re_{tf}, even for experiments performed in the same wind-tunnel, the attendant dimensional amplification rates at equal boundary layer thickness should be in the ratio $(\alpha^*_{ic}/\alpha^*_{if}) \ll 1$. However, theory (i.e. the afore-noted approximate transformation of the stability equations) as well as experiments (Fig. 11) indicate that $(\alpha^*_{ic}/\alpha^*_{if}) \simeq 1$. One is then led to conjecture that the observed $Re_{tc} > Re_{tf}$ result from a substantively longer non-linear stage of transition on cones as compared to flat plates. This view, qualitatively supported by the measurements discussed above (Figs. 1 through 3), implies that the systematic correlation/extrapolation of transition results/predictions among different vehicle configurations depend on a case-by-case sequential analysis of the linear and non-linear stages of the process. In accord with the evidence of Refs. 15, 16 and 26,

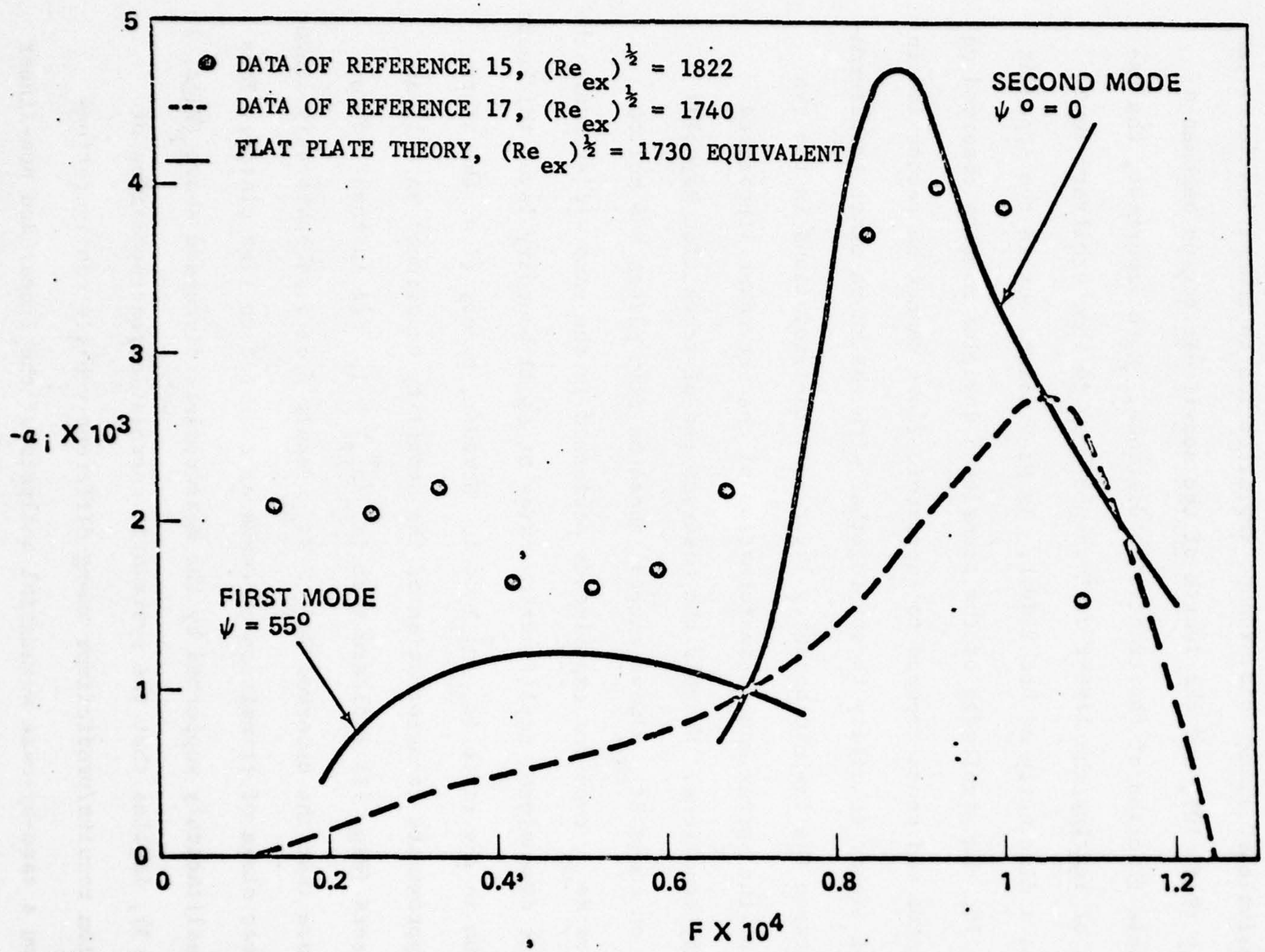

Fig. 11 Comparison between amplification rates measured on slender cones at $M_e \simeq 7, (T_w/T_o) \simeq .7$ (Refs. 15, 17) and flat plate predictions adjusted for equivalent Re_{ex}. From Reference 15.

transition is associated with an instability of the dominant equilibrated non-linear "laminar waves", i.e. a process whose nature and rate can only be ascertained upon a systematic description of the presently unknown flow structure attendant thereto. Such instability of finite amplitude wave-trains appears not only relevant to "natural" transition, but also to the practically interesting problem (perhaps prevalent on RV heat shields) of transition induced by three-dimensional roughness elements; in fact, strikingly similar instantaneous velocity distributions at wave breakdown, followed by hairpin loop generation, are observed in the two cases, according to Ref. 26. Thus, the study of non-linear critical layer effects and attendant non-linear wave structures promises to shed light on several aspects of practical transition prediction and design problems.

III. OUTLINE OF A NON-LINEAR STABILITY MODEL

The model outlined here laregly exploits the results of a systematic investigation of non-linear, critical-layer-controlled, waves being carried out under a separate AFOSR-sponsored effort. The premise of that effort resides in the recent experimental evidence about the presence in all turbulent flows of quasi-ordered, large scale events/structures, which occur randomly, but with statistically definable mean periods (Ref. 14). According to that evidence the nature, growth, equilibration and subsequent agglomeration and/or modulation of the structures vary from family to family of flows; however, for a given family (say incompressible flat plate boundary layers), they show little dependance on Reynolds number throughout the non-linear transitional and fully developed turbulent regimes. In both regimes the evolving structures appear to dominate the macroscopic aspects of the flow, including the rate of growth/entrainment and the Reynolds stresses, as well as the process of intermittent turbulence production by three-dimensional energy cascade connected with high wave-number instabilities evolved within, and amplified by, the structures themselves. The structures also exhibit throughout a remarkable degree of coherence such as pertains to non-linear wave-trains possessing non-dispersive characteristics, i.e. trains whose spectral components propagate as bound waves not obeying the linear dispersion relation. Such behavior can hardly represent the result of non-linear interactions among the resonant components of a fluctuations spectrum made up of many linear random dispersive free waves with Gaussian or near-Gaussian statistics; however, it is in principle consistent with the evolution of non-linear, critical-layer-controlled, wave-trains whose spectra are

made up of bound-wave components.

Among the many experimentally observed structures those associated with two-dimensional homogeneous incompressible free mixing layers are perhaps the best documented ones in both the non-linear transitional (Ref. 30) and the turbulent regimes (Refs. 31, 32). As those structures are also largely two-dimensional, they constituted an obvious first target for the theoretical investigations under the aforementioned AFOSR sponsored effort. By fortunate coincidence the "laminar waves" observed in Refs. 15 and 16 are also two-dimensional; hence, some of the theoretical results could readily be extrapolated to their analysis as presented below.

The motion of an incompressible free shear layer is inviscidly unstable and governed by the vorticity conservation equation. For the strictly two-dimensional problem considered here, i.e. a basic parallel flow with velocity $(\bar{U}, 0, 0)$, vorticity $(0, 0, \Omega)$ and Reynolds number Re (based on average mean flow velocity $\bar{U}$ and flow width L), plus a superposed perturbation of small amplitude A involving velocities (u, v, 0) and vorticity $(0, 0, \omega)$, the equation is

$$\frac{\partial \omega}{\partial t} + (\bar{U} - c)\frac{\partial \omega}{\partial x} + v\frac{\partial \Omega}{\partial y} = - A\left(u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y}\right) + \frac{1}{ARe}\nabla(\Omega + A\omega) \qquad (3)$$

in a frame of coordinates moving with the phase velocity c of the disturbance. Whereas the solutions of classical linear inviscid stability theory are obtained by setting to zero the right hand side of equation (3), the approximation must fail to higher order whenever one of the neglected terms becomes comparable to the terms retained. This occurs for $(\bar{U} - c) \to 0$ near the critical point $(y = y_c)$. In that neighborhood the correct vorticity balance

must take into account the dominant terms on the right hand side of equation (3). A uniformly valid solution to the equation must accordingly be evolved in the context of classical matched asymptotic expansion techniques, with the expansion parameter selected in accordance with the predominant term on the right hand side.

In classical stability theory the viscous term at the right hand side of equation (3) is assumed dominant, independent of wave amplitude and, to the leading order, the outer linear inviscid solution is matched with an inner viscous solution valid in a region of extent $[(\alpha \text{ Re } u_c')^{-1/3} L]$ about the critical layer. However, as noted in Section II, the assumption of a dominant viscous term is only correct for modest disturbance amplitudes A, typically $A \lesssim 10^{-2}$ for $Re \simeq 10^3$ and α as well as u_c' about unity. For larger amplitudes the non-linear vorticity transport $Av(\partial\omega/\partial y)$ dominates the right hand side of (3). The matched asymptotic expansion technique then shows that, for a spatially growing wave of amplitude A, wave number α_r, growth rate $(-\alpha_i)$ and frequency β, the noted non-linear term contributes measurably to the vorticity balance within an (inner) region of lateral extent $[(A/u_c')^{\frac{1}{2}} L] = [(A_o/u_c')^{\frac{1}{2}} \exp(-\frac{\alpha_i}{2} x) L]$ about the critical layer. Under those conditions the flow obtained by the superposition of a small but finite-amplitude two-dimensional wave upon a basic parallel flow, is characterized by a stream function ψ in wave fixed coordinates having matched inner and outer expansions, respectively, of the form

$$\psi^{(i)}(x,y,t) = \sum_{m=0,1,..} A^{1+(m/2)} \Phi_m \left[\xi, \eta \exp\left(-\frac{\alpha_i}{\alpha_r}\xi\right), \epsilon\right] \tag{4a}$$

$$\psi^{(o)}(x,y,t) = \int_{y_c}^{y} (U-c)\, dy + \sum_{n=0,1..} A^{(n/2)} \varphi_n [y,\xi] \tag{4b}$$

where

$$\xi = (\alpha_r x - \beta t) \tag{5a}$$

$$\eta = \left[\frac{u'_c}{A} \frac{(y-y_c)^2}{2} + \cos \xi\right] \tag{5b}$$

and

$$\epsilon = |\alpha_i \alpha_r^{-2} \beta (Au'_c)^{-\frac{1}{2}}| \tag{5c}$$

is a growth parameter manifested in the inner equations. Typically, for $A \gtrsim 10^{-2}$, $\epsilon < 1$ so that the inner solution may be sought by asymptotic expansion in powers of ϵ*. Upon use of Lighthill's method of stretched coordinated this expansion takes the form

$$\eta = \zeta + \sum_{p=1,2,..} \epsilon^p \zeta_p (\xi,\zeta) \tag{6a}$$

$$\Phi_m \left[\xi, \eta \exp\left(-\frac{\alpha_i}{\alpha_r} \xi\right), \epsilon\right] = \sum_{p=0,1,..} \epsilon^p \Phi_{mp}\left[\xi, \zeta \exp\left(-\frac{\alpha_i}{\alpha_r} \xi\right)\right] \tag{6b}$$

and the functions Φ_{mp} can be determined consistent with the inner equations as well as with the requirement that the attendant solution be periodic in ξ. Detailed matching in the overlap domain $(y-y_c) \to 0$, $\zeta \to \infty$, then yields

$$\varphi_n [y,\xi] = \sum_{q=0,1,..} \varphi_{nq} [y] \cos\left(q \frac{\xi}{2}\right) \tag{6c}$$

* Robinson (Ref. 33) has pursued a similar approach for marginally unstable temporally growing waves. However, he has obtained erroneous results for the amplification rates due to improper evaluation of the vorticity in the inner region. Also he could not find solutions for the practically interesting case of spatially amplifying waves due to improper choice of the independent variables in the inner region.

By this process the effect of non-linearity (viz, transversal transport of perturbation vorticity near the critical layer) in forcing mean flow perturbations, harmonics and subharmonics in the outer region becomes manifest. Specifically one finds that:

INNER SOLUTIONS		MATCH/FORCE	OUTER SOLUTIONS	
m	p		n	q
0	0		2	2
1	0		1	0
			3	1,2,4,..

etc. Thus for an amplifying two-dimensional finite amplitude wave in incompressible flow, non-linear critical layer effects result in the generation of bound-wave harmonics and subharmonics having growth rates 1.5 times as large as that of the fundamental. These results, which nicely reproduce the observations of Ref. 30 for the non-linear stages of transition in a mixing layer, represent a direct extension of those set forth in Refs. 28, 29 for neutral non-linear waves. In fact the methods of analysis are essentially the same, except for the use of the parameter ϵ in lieu of the parameter $\lambda = [(\alpha\, Re_\delta\, u_c')\,(A/u_c')^{3/2}]^{-1} \ll \epsilon$ in the asymptotic expansion of the inner solutions. Thus, the findings of Refs. 28, 29 for neutral waves, as well as their extension to slowly varying non-linear waves (Ref. 34), may readily be transposed to the amplifying and equilibrating finite amplitude disturbances encountered in the transitional flows discussed in this report. In that connection we specifically address below the finding by Demetriades of "... a complex stability diagram with at least three unstable regions. The lower of these regions appears to be generated by the first and second

mode instabilities, but no theoretical guidance exists to help identify the contributing modes for the other regions" (see Refs. 16, 17 and Fig. 4).

We begin by noting the results of Ref. 28 for a single oblique neutral finite-amplitude wave propagating at an angle θ to an incompressible parallel flow U(y). Working in wave-oriented coordinates Benney and Bergeron show that the strictly two-dimensional solution still describes the flow in planes parallel to the wave vector, but the cross flow perturbation velocity

$$w = (U' \sin \theta)\,(U \cos \theta - c)^{-1}\, \varpi_{22}(y) \tag{7a}$$

becomes singular at the critical layer; specifically, for $y \to y_c$

$$w = \tan \theta \left[\frac{P_2(y-y_c)}{(y-y_c)} + \frac{U''_c}{U'_c} P_1\,(y-y_c) \log\,(y-y_c) + C\,P_1\,(y-y_c) \right] \tag{7b}$$

where

$$P_i\,(y-y_c) = 1 + \sum_{m=1,..} a_{im}\,(y-y_c)^m \qquad (i = 1,2)$$

and C is a multiplying constant. This singularity is removed by introducing an inner expansion

$$W = (U \cos \theta - c) \tan \theta + A\,w = A^{\frac{1}{2}}\, W_i^{(o)} + \ldots \tag{8}$$

which leads to a solution $W_i^{(o)}$ having harmonics as well as subharmonics in the non-linear critical layer.

We then pass to consider the solutions of linear inviscid stability theory for a parallel compressible flow characterized by mean velocity and temperature profiles U(y), T(y). As is well known (Ref. 35), the v perturbation velocity near the critical layer is described by generalized Tollmien

solutions formally identical to those encountered in incompressible flow, but the temperature perturbation θ acquires the singular behavior

$$\theta \sim \frac{1}{(y-y_c)} + \frac{T_c}{U_c'}\left[\frac{d}{dy}\left(\frac{U'}{T}\right)\right]_{y_c} \log (y-y_c) + \ldots \tag{9}$$

formally identical to that found for the cross flow perturbation velocity w in equation (7b). This singularity is also removed by introducing an inner expansion

$$\mathcal{T} = T + A\theta = A^{\frac{1}{2}} \mathcal{T}_i^{(o)} + \ldots \tag{10}$$

again to obtain a solution $\mathcal{T}_i^{(o)}$ characterized by the generation of harmonics as well as a subharmonic in the non-linear critical layer. Thus, we find that, upon attaining finite amplitude and becoming controlled by critical layer effects, an amplifying two-dimensional wave in compressible flow must exhibit a bound-wave spectrum of temperature fluctuations composed of the fundamental frequency as well as its harmonics, the latter growing at one half the rate of the fundamental. This is exactly the situation displayed in Fig. 4, where: 1) the modes populating the puzzling middle unstable region can be associated with second harmonics of those lower region disturbances which, at the measurement station, have finite amplification rate ($F \lesssim 1.4 \times 10^{-4}$) and, presumably, amplitude above the threshold for non-linear critical effects ($F \gtrsim 1.1 \times 10^{-4}$); 2) the amplification rates of the middle region modes are approximately one half those of the associated fundamental (lower region) modes; 3) the upper unstable region may reasonably be connected with the third harmonics of the same lower region disturbances. The interpretation of Demetriades' results in terms of non-linear critical layer effects, already suggested in Section II, is then forcefully supported, and the role

of such effects in transition data analysis and predictions is confirmed.

The initial understanding and modeling of non-linear flow processes presented above suggests that distinct progress toward rational and systematic transition prediction criteria may be achieved by further coherent theoretical and experimental investigations of those processes. On modeling grounds only a first step has been achieved as a consistent, fundamentally based, quantitative description of the mechanisms controlling the sequential equilibration, breakdown and/or modulation of finite-amplitude, critical-layer-controlled wave-structures remains to be accomplished. In that connection only speculative lines of approach and interpretation can be offered at present; some are submitted in the following section.

IV. CONCLUDING REMARKS

On the premise that the forced response/stability viewpoint constitutes the most satisfying and promising conceptual approach to transition analysis and prediction, we have used it as the framework for a critical review of data from selected low - as well as high-speed laboratory experiments where the evolving disturbance spectrum within transitional boundary layers has been measured by hot wire anemometry. Upon this review we have determined that the growth of unstable disturbances becomes influenced by non-linear effects at unsuspectedly low amplitudes, typically $A \simeq 10^{-2}$; thus, the non-linear stages of transition, e.g. the "laminar waves", occupy a considerable fraction of the transition distance, especially for wind-tunnel experiments at supersonic velocities where relatively modest disturbance amplifications (by less than a factor 100) are measured between critical and transition Reynolds numbers. Under these conditions linear theory becomes inadequate for accurate data analyses and judicious extrapolations, and realistic non-linear models must be employed. By critical review of data vis-a-vis existing models, as well as by order of magnitude considerations applied to the full flow equations, we have argued that the observed non-linearities largely manifest non-linear critical layer effects as opposed to weakly non-linear interactions among random dispersive linear waves. We have stated a model descriptive of such critical layer effects in strictly two-dimensional flows and we have shown that it qualitatively predicts the frequency/amplification characteristics of the modes populating the upper unstable regions in the complex stability diagram determined experimentally by Demetriades.

On the basis of the arguments/findings set forth above and the available experimental evidence we suggest that further coordinated analytical and experimental investigations of the two and three-dimensional flow structures associated with non-linear critical layer effects may yield a quantitative understanding of their tendency to equilibrate and/or evolve intrinsic instabilities which, in turn, may cause the birth of turbulent spots as well as the subsequent modulated recurrence of coherent structures in the turbulent regime. Subject to favorable quantitative stage-by-stage comparisons between theoretical developments and controlled experimental observations, such inquires should not only provide a systematic attack on many current transition dilemma,but also a rational basis for evolving/extending/qualifying transition data correlations and attendant design criteria.

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